Reliability Allocation

Amnon Ganot - July 2012

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Outline

- Reliability Definition
- Reliability Matrix
- Reliability Block Diagram - RBD
- Reliability Allocation Methods
- Summary

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Overview

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Reliability Definition

- The **probability** that equipment will perform its intended function (**mission**), within stated conditions, for a specified period

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Reliability Matrix

"You cannot manage what you cannot control, and you cannot control what you cannot measure"

Source unknown

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Reliability Matrix

- By quantifying reliability metrics, we can measure and define a target value.
- Meeting or exciding the target value is then our product reliability objective

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Reliability Function

- Reliability Function
  - The Probability of component/system surviving a time **t**.
  - Alternatively, the number of units surviving at time **t** divided by the initial number of units.

$$R(t)$$

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Reliability Function

time independent

- Example
  - A nuclear submarine successfully launched a rocket 1,500 miles down the Atlantic test range to chalk up the 43^{rd} success in the last 45 firing

$$R(t) = \frac{43}{45} = 0.96$$

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Reliability Function

time dependent

- Example
  - 200 identical products are being reliability tested for 40 hours with the following results:
    - 10 fail just before completing 10 hours of satisfactory operation
    - 5 fail at 20 hours
    - 2 at 30 hours
    - and 3 at 40 hours

$$R(10)=\frac{190}{200}=0.95$$

$$R(20)=\frac{185}{200}=0.925$$

$$R(30)=\frac{183}{200}=0.915$$

$$R(40)=\frac{180}{200}=0.9$$

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Failure Rate

$$\lambda(t)$$

- Instantaneous failure rate, hazard rate, or just the failure rate:
  - Probability of failure in unit time of a device that is still working.
  - The instantaneous rate of failures for devices of population that have survived to time **t**.
- The most common reliability metrics

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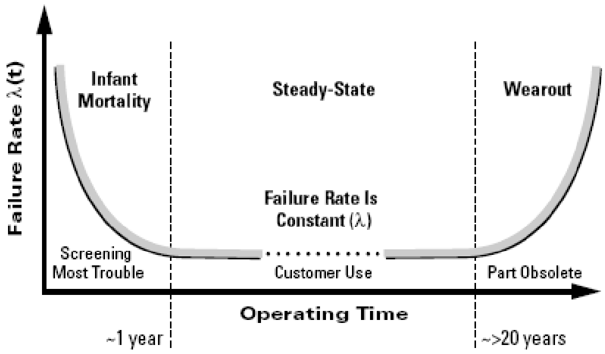
Failure Rate

$\lambda(t)$

- The failure rate itself is either time dependent or time-independent



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Failure Rate

- Component Reliability (Discrete)
  - Resistors, capacitors, diodes, ICs, etc.
- System Reliability (Hybrids & Assemblies)
  - Usually, the whole is equal to the sum of the parts for the failure rate.
  - *Example: Reliability of a light bulb*
    - Failure rate = λ system
    - λ system = λ filament + λ seal + λ connections
  - The whole is not equal to the sum of the parts when there is redundancy (double filament inside).

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MTBF / MTTF

- MTBF/MTTF Definition
  - Mean productive Time Between *equipment-related Failures*
  - Mean productive Time To *equipment-related Failures*
  - Productive Time
    - The time when the equipment is performing its intended function
  - Equipment Related Failures
    - Any unplanned event that changes the equipment to a condition where it cannot perform its intended function solely caused by the equipment

MTBF / MTTF

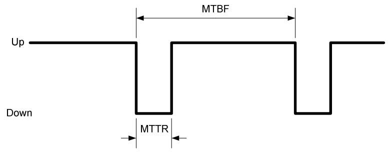
- MTBF is the preferred term instead of MTTF when repairs are involved.
- Both are the inverse of the failure rate when the failures rate is constant

$$MTBF = \frac{1}{Constant\ Failure\ Rate}$$

$$MTBF = \frac{1}{\lambda}$$

MTTR

- Weighted average time to repair *equipment-related failures*; the average time to correct an *equipment-related failure* and return the equipment to a condition where it can perform its intended function



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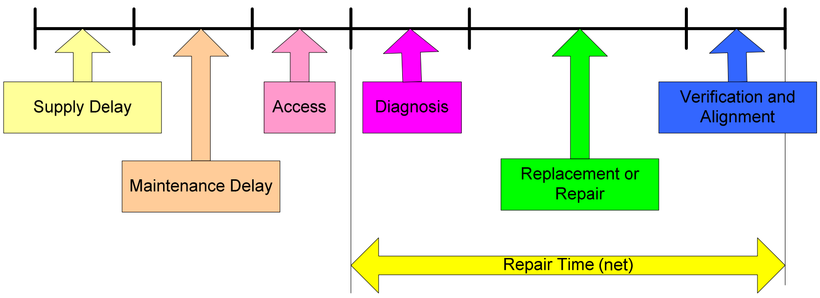
MTTR - Timeline



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Availability - Types

- ***Inherent Availability***
  - This is the ideal state of availability
  - The only considerations are the MTBF (reliability) and the MTTR (Maintainability).
  - This measure does not take into account the time for preventive maintenance and assumes repair begins immediately upon failure of the system

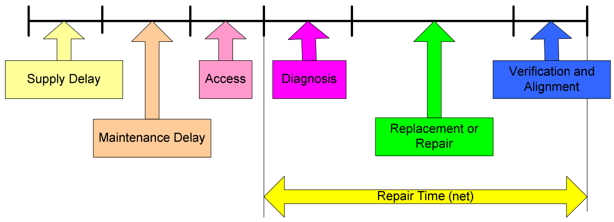
$$A_I = \frac{MTBF}{MTTR + MTBF}$$

Availability - Types

- ***Achieved Availability***
  - Achieved Availability is somewhat more realistic in that it takes preventive maintenance into account as well as corrective maintenance
  - The assumption here is that, as in Inherent Availability, there is no loss of time waiting for the maintenance action to begin

Availability - Types

- ***Operational Availability***
  - This is what generally occurs in practice
  - Operational Availability takes into account that the maintenance response is not instantaneous, repair parts may not be in stock as well as other logistics issues



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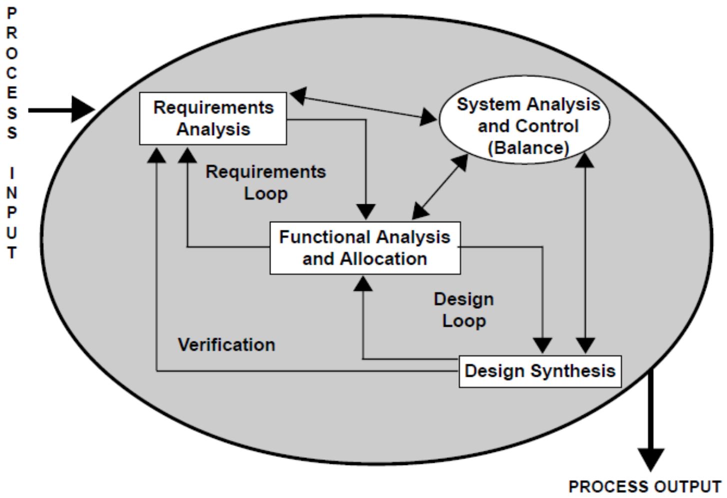
Reliability Allocation Methods

System Engineering Process



Reliability Allocation

- Reliability Allocation deals with the setting of reliability goals for individual subsystems such that a specified system reliability goal is met and the subsystem goals are well balanced among themselves
- Well-balanced usually refers to the minimization of overall development cost (both BOM & NRE) while maintaining the target system reliability

Reliability Allocation

- Reliability allocations for hardware/software systems can be started as soon as the system reliability models have been created (RBD)
- For example, if you are building a system made up of five different subassemblies, and you have a known MTBF goal for the entire system, you can allocate, or split up, the MTBF objectives for each of the five components in a well-balanced way that results in meeting your established overall goal for the system

Reliability Allocation

- This may be especially useful in situations where different groups, or even different subcontractors, are responsible for certain subassemblies
- The apportionment of reliability values between the various subsystems and elements can be made on the basis of complexity, criticality, estimated achievable reliability, or any other factors considered appropriate by the analyst making the allocation

Reliability Allocation

- System-level allocations are successively decomposed using the reliability model(s) until an appropriate set of reliability measures has been apportioned to each hardware and hardware/software component of the system
- The allocation of reliability values to lower-tiered hardware elements is a continuation of the allocation process begun at the system level

Reliability Allocation

- The system level hardware reliability models are used to successively apportion the required reliability measures among the various individual pieces of hardware and from the hardware equipment level to the various internal elements
- For existing hardware items, the reliability allocations used should be based on the reliability performance of previously produced equipment

Reliability Allocation

- As a system integrator, you can specify the MTBF goals you want each subassembly to achieve
- Thus, a key element of the allocation process is to determine how best to allocate MTBF requirements in a well-balanced way across your entire system
- This is where reliability allocation methods play a vital role
- By considering the various alternatives for computing reliability allocation goals, you can select the method that best suits your needs

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Reliability Allocation

- The first step in the allocation process is to describe the system Reliability Block Diagram (RBD)

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Reliability Block Diagram - RBD

Reliability Block Diagram - RBD

- A *Reliability Block Diagram* (RBD) is used to performs the system reliability, maintainability and availability analysis on large and complex systems using block diagrams to show network relationships

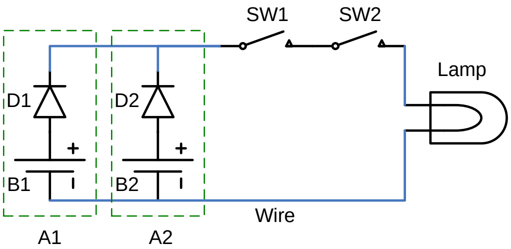
Reliability Block Diagram - RBD

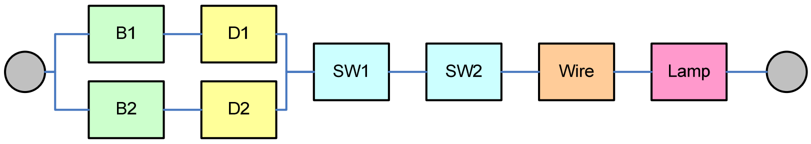
- The structure of the *Reliability Block Diagram* defines the logical interaction of failures within a system that are required to sustain system operation (specific mission or function)

Reliability Block Diagram - RBD

- A *Reliability Block Diagram* (RBD) is an event diagram
- It answers the following question:
  - Which element of the item under consideration are necessary for the fulfillment of the required function and which can fail without affecting it?
- The elements which are necessary for the required function are connected in *series*, while elements which can fail with no effect on the required function (redundancy) are connected in *parallel*
- ***Each required function might have its own*** ***Reliability Block Diagram***

RBD - Example





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RBD - Data

- Each block represents the relevant unit's failure function. The source for the failure function can be one of the followings:
  - Allocated Value
  - Field Data
  - Test Data (Reliability Lab)
  - Standards (e.g., MIL-HDBK-217, Bellcore, etc.)

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Reliability Allocation Methods

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Reliability Allocation Methods

- When developing a new product or improving an existing one, engineers are often faced with the task of designing a system that must meet a certain set of reliability specifications
- This involves a balancing act in order to determine how to allocate reliability among the subsystems/components in the system

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Reliability Allocation Methods

- Reliability allocation involves solving the following inequality:

$$f(R_1, R_2, \dots \dots . R_n) \geq R_s$$

- Where:
  - R_i is the reliability allocated to the i^{th} subsystem/componenet
  - f is the functional relationship between the subsystem/component and the system
  - R_s is the required system reliability

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Reliability Allocation Methods

- Several Algorithms for reliability allocation have been developed:
  - Equal Apportionment
  - AGREE
  - ARINC
  - Feasibility of Objectives Apportionment
  - Reparable Systems Apportionment
  - Cost/Penalty Function

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Equal Apportionment

- The simplest apportionment technique is to distribute the reliability uniformly among all components
- Equal apportionment assumes a series of n subsystems, all in series and having an exponential failure distribution
- Each subsystem is assigned the same reliability

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Equal Apportionment

- The mathematical model can be expressed as:

$$R_s = \prod_{i=1}^{n} R_i$$

or

$$R_i = (R_s)^{1/n}$$

- Where:
  - R_s is the system reliability goal
  - R_i is the reliability allocated to the i^{th} subsystem
  - i is the subsystem index
  - n is the total number of subsystems

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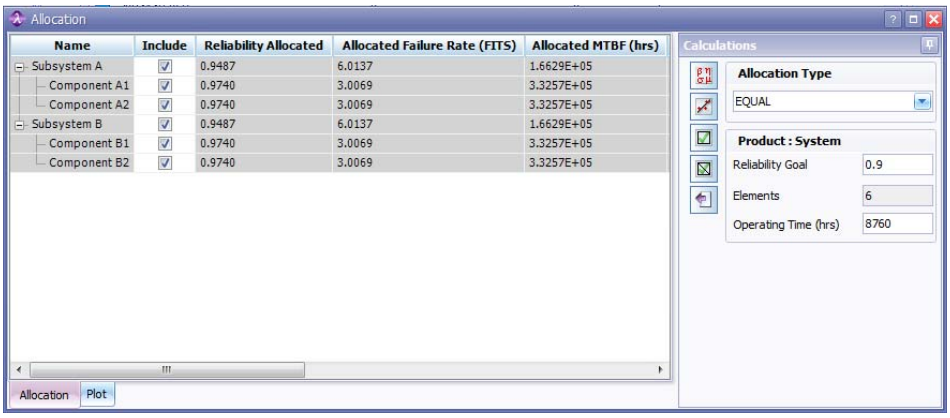
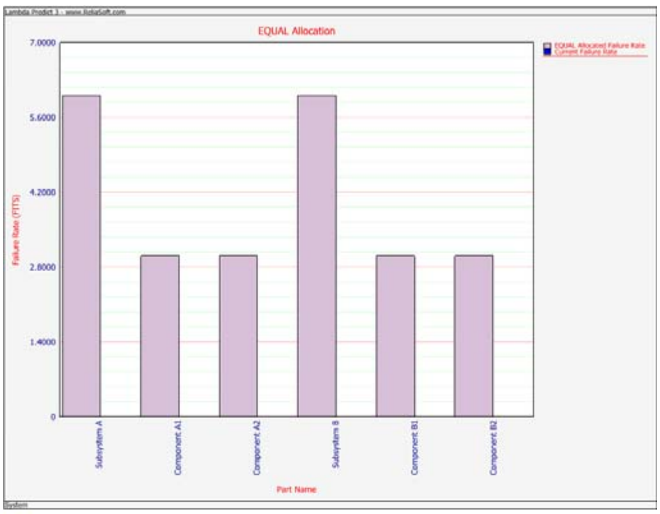
Equal Apportionment - Example



Allocation

Name	Include	Reliability Allocated	Allocated Failure Rate (FITS)	Allocated MTBF (hrs)
Subsystem A	☑	0.9487	6.0137	1.6629E+05
Component A1	☑	0.9740	3.0069	3.3257E+05
Component A2	☑	0.9740	3.0069	3.3257E+05
Subsystem B	☑	0.9487	6.0137	1.6629E+05
Component B1	☑	0.9740	3.0069	3.3257E+05
Component B2	☑	0.9740	3.0069	3.3257E+05

Equal Apportionment - Example



AGREE Apportionment

- The AGREE apportionment method, designed by the Advisory Group on Reliability of Electronic Equipment, determines a minimum acceptable mean life for each subsystem in order to fulfill a minimum acceptable system mean life
- The AGREE method assumes that all subsystems are in series and have an exponential failure distribution
- This method takes into account both the complexity and the importance of each subsystem

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AGREE Apportionment

- The mathematical model is:

$$MTBF_i = nw_it_i/n_i[-lnR_s(t)]$$

and

$$R_i(t_i) = e^{-t_i/MTBF_i}$$

- Where:
  - $R_s(t)$ is the system reliability
  - $R_i(t_i)$ is the allocated reliability for the i^{th} subsystem
  - t is the system operating time
  - t_i is the operating time of the i^{th} subsytem
  - i is the subsystem index
  - w_i is the importance factor for the i^{th} subsystem
  - n_i is the number of sub-elements for the i^{th} subsystem
  - n is the total number of sub-elements, which is given by $\sum_{i=1}^{k} n_i$

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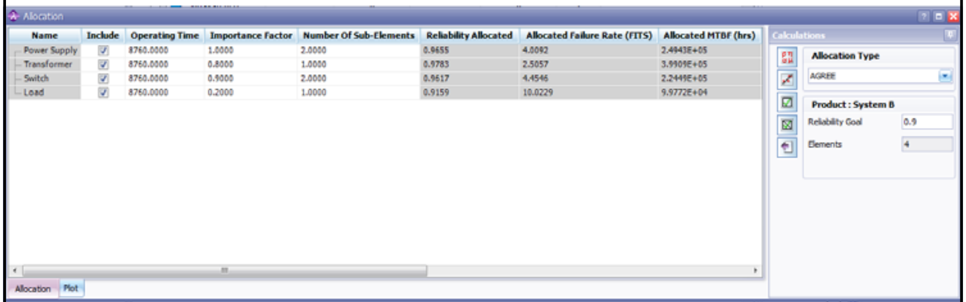
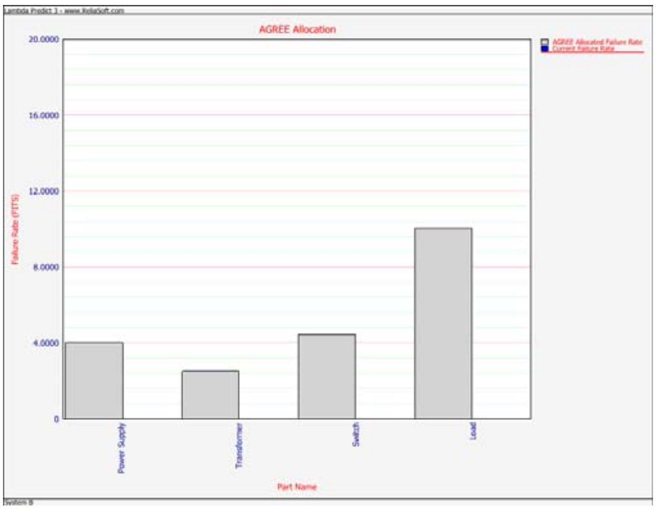
AGREE Apportionment - Example



Name	Include	Operating Time	Importance Factor	Number Of Sub-Elements	Reliability Allocated	Allocated Failure Rate (FITS)	Allocated MTBF (hrs)
Power Supply	✓	8760.0000	1.0000	2.0000	0.9655	4.0092	2.4943E+05
Transformer	✓	8760.0000	0.8000	1.0000	0.9783	2.5057	3.9909E+05
Switch	✓	8760.0000	0.9000	2.0000	0.9617	4.4546	2.2449E+05
Load	✓	8760.0000	0.2000	1.0000	0.9159	10.0229	9.9772E+04

AGREE Apportionment - Example



ARINC Apportionment

- The ARINC apportionment method, designed by the Advisory Group on Reliability of Electronic Equipment, assumes that all subsystems are in series and have an exponential failure distribution
- From the present allocation of the subsystems, allocation improved system failure rates are derived based on weighting factors

ARINC Apportionment

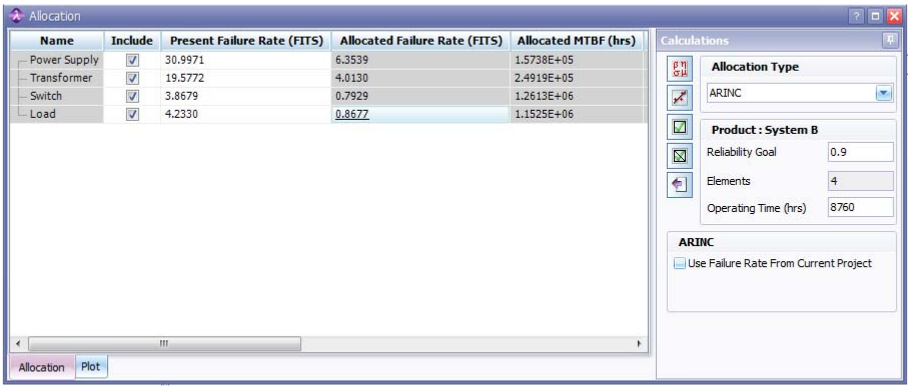
- The mathematical expression is:

$$w_i = \lambda_i / \sum_{i=1}^{n} \lambda_i$$

$$\lambda'_i = w_i \lambda_s$$

- Where
  - n is the total number of subsystems
  - λ_i is the present failure rate of the i^{th} subsystem
  - λ_s is the required system failure rate
  - λ'_i is the failure rate allocated to the i^{th} subsystem

ARINC Apportionment - Example



Name	Include	Present Failure Rate (FITS)	Allocated Failure Rate (FITS)	Allocated MTBF (hrs)
Power Supply	☑	30.9971	6.3539	1.5738E+05
Transformer	☑	19.5772	4.0130	2.4919E+05
Switch	☑	3.8679	0.7929	1.2613E+06
Load	☑	4.2330	0.8677	1.1525E+06

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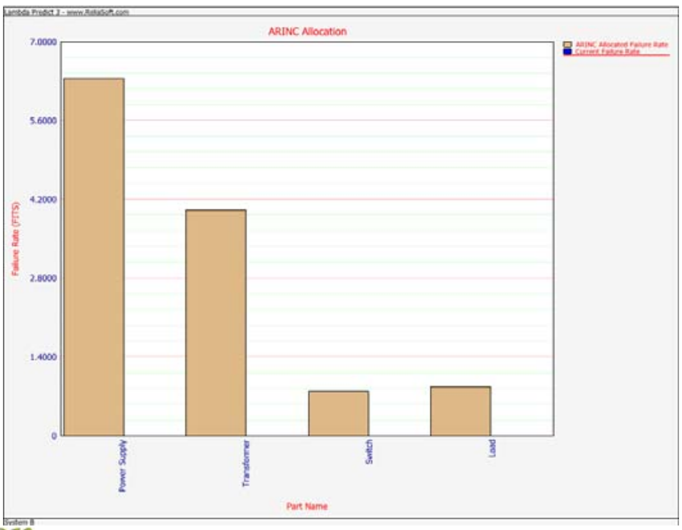
ARINC Apportionment - Example



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Feasibility of Objectives

- Feasibility of Objectives apportionment is based on numerical ratings of the designs state of the art, the system complexity, the mission operating time and the environment for each item to which the product reliability will be allocated, assuming that all subsystems are in series and have an exponential failure distribution

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Feasibility of Objectives

- Ratings for each factor range from a low of 1 to a high of 10
- These four criteria ratings are multiplied together to get an overall weighting and are normalized so that the product sum is 1

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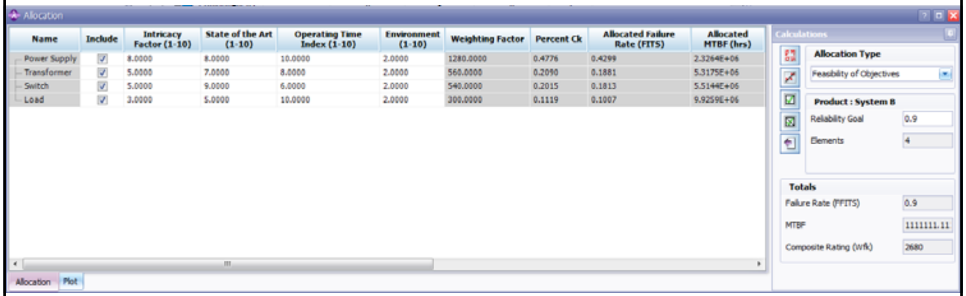
Feasibility of Objectives

- The mathematical model can be described as:
  - $W_i = r_{i1} \; x \; r_{i2} \; x \; r_{i3} \; x \; r_{i4}$
  - $W = \sum_{i=1}^{N} W_i$
  - $C_i = \frac{W_i}{W}$
  - $\lambda_i = C_i \lambda_s$
  - $\lambda_s T = \sum \lambda_i T$

Feasibility of Objectives

- Where:
  - T is the operating duration
  - λ_s is the system failure rate
  - λ_i is the allocated subsystem i failure rate
  - C_i is the percent weighting factors of the i^{th} subsystem
  - W_i is the composite rating for the i^{th} subsystem
  - N is the total number of subsystems
  - r_{ik} is the k^{th} rating result for the i^{th} subsystem

Feasibility of Objectives - Example



Allocation

Name	Include	Intricacy Factor (1-10)	State of the Art (1-10)	Operating Time Index (1-10)	Environment (1-10)	Weighting Factor	Percent Ck	Allocated Failure Rate (FITS)	Allocated MTBF (hrs)
Power Supply	☑	8.0000	8.0000	10.0000	2.0000	1280.0000	0.4776	0.4299	2.3264E+06
Transformer	☑	5.0000	7.0000	8.0000	2.0000	560.0000	0.2090	0.1881	5.3175E+06
Switch	☑	5.0000	9.0000	6.0000	2.0000	540.0000	0.2015	0.1813	5.5144E+06
Load	☑	3.0000	5.0000	10.0000	2.0000	300.0000	0.1119	0.1007	9.9259E+06

Repairable System Apportionment

- Repairable Systems apportionment allocates subsystem failure rates to allow the system to meet an availability objective for a repairable system
- This technique assumes all subsystems to be in series, with exponential failure distributions and constant repair rates
- By determining the ratio of the allocated failure rate to the repair rate for each subsystem based on a steady-state availability calculation, the failure rate allocated to each subsystem can be determined

Repairable System Apportionment

- The math expression of this method is:

$$A_i = (A_s)^{1/n}$$
$$\theta_i = \frac{1}{A_i} - 1$$
$$\lambda_i = \frac{\theta_i}{MTTR_i}$$

- Where:
  - A_s is the required system Availability
  - A_i is the allocated availability to the i^{th} subsystem
  - n is the total number of subsystems
  - θ_i is the ratio of the allocated failure rate to the repair rate for the i^{th} subsystem
  - $MTTR_i$ is the mean time to repair of the i^{th} subsystem
  - λ_i is the allocated failure rate for the i^{th} subsystem

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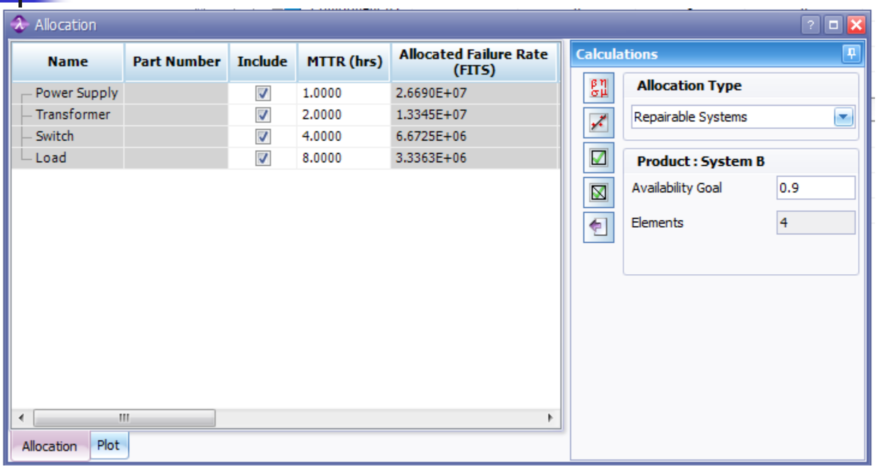
Repairable System Apportionment - Example



Name	Part Number	Include	MTTR (hrs)	Allocated Failure Rate (FITS)
Power Supply		☑	1.0000	2.6690E+07
Transformer		☑	2.0000	1.3345E+07
Switch		☑	4.0000	6.6725E+06
Load		☑	8.0000	3.3363E+06

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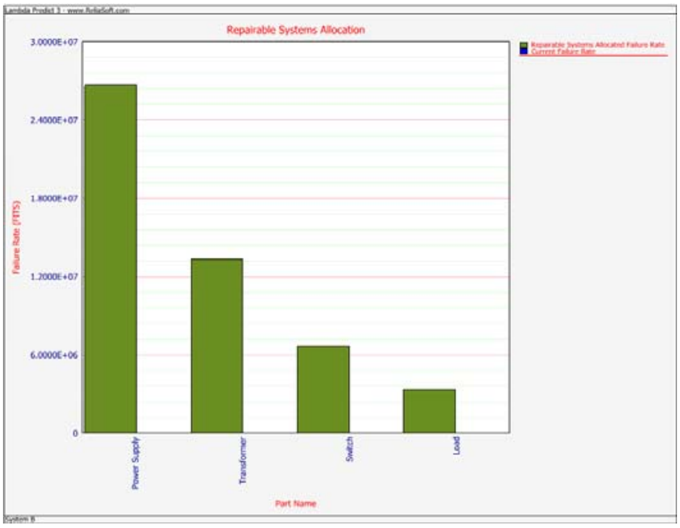
Repairable System Apportionment - Example



Cost/Penalty Function

- There is always a cost associated with changing a design, due to change of vendors, use of higher-quality materials, retooling costs, administrative fees, or other factors
- Before attempting to improve the reliability, the cost as a function of reliability for each component must be obtained
- Otherwise, the design changes may result in a system that is needlessly expensive or over-designed

Cost/Penalty Function

- Developing the "cost of reliability" relationship will give the engineer an understanding of which components/subsystems to improve and how to best concentrate the effort and allocate resources in doing so
- The first step will be to obtain a relationship between the cost of improvement and reliability

Cost/Penalty Function - Example

- An exponential behavior for the cost is assumed, and the function has the following form:

$$C_i(R_i) = e^{(1-f)*\frac{R_i - R_{min,i}}{R_{max,i} - R_i}}$$

- Where:
  - $C_i(R_i)$ is the penalty function (or cost) as a function of component reliability
  - f is the feasibility of improving a component's reliability relative to other components in the system
  - $R_{min,i}$ is the current reliability at the given mission time at which the optimization is to be performed
  - $R_{max,i}$ is the maximum achievable reliability at the given mission time at which the optimization is performed

Summary

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Summary

- Establishing reliability goals during system design is critical to ensuring that your overall reliability objectives will be achieved
- It is important, therefore, to appropriately allocate MTBF goals across all your system components in the most effective manner
- Selecting the appropriate reliability allocation technique is a critical part of this effort
- The technique you employ should be selected based on the information available about your system and your overall requirements

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Questions?

Thanks for your attention

